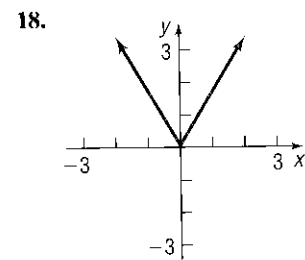
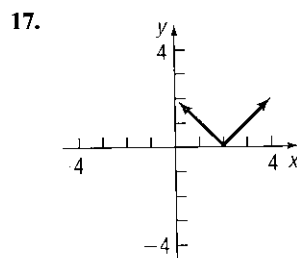
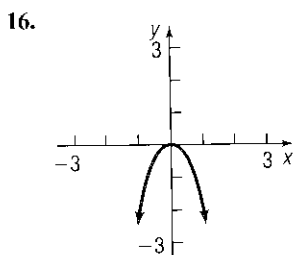
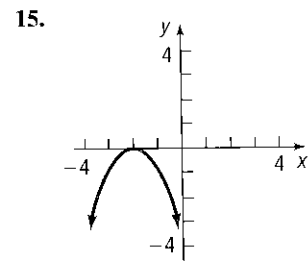
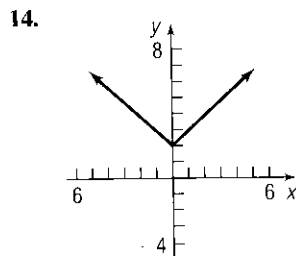
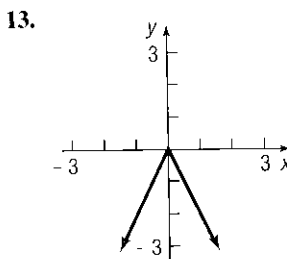
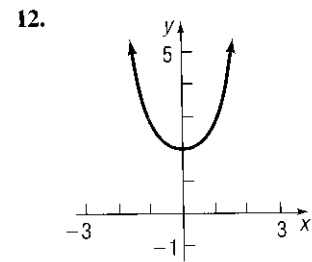
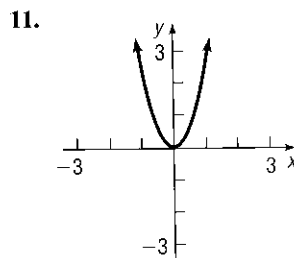
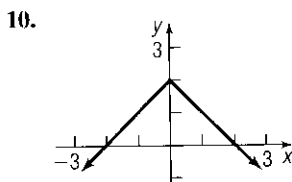
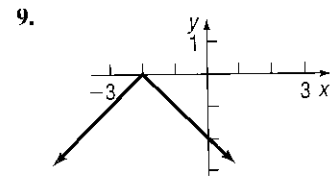
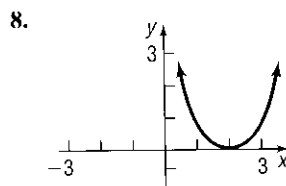
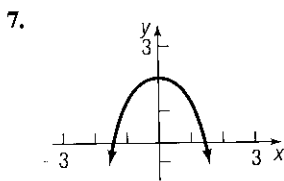


Final Exam: Review Problems

- Graph using transformations.
- Operations with functions.
- Composite functions.
- Polynomial functions.
- Rational functions.
- Systems of equations.

1. Match each graph to one of the following functions.

- | | | | |
|-------------------------------|--------------------------------|------------------|-------------------|
| a) $y = x^2 + 2$ | d) $y = -x^2 + 2$ | g) $y = x + 2$ | j) $y = - x + 2$ |
| b) $y = \left(x - 2\right)^2$ | e) $y = -\left(x + 2\right)^2$ | h) $y = x - 2 $ | k) $y = - x + 2 $ |
| c) $y = 2x^2$ | f) $y = -2x^2$ | i) $y = 2 x $ | l) $y = -2 x $ |



2. Graph each function using transformation techniques.

- | | | |
|--------------------------------------|--|---|
| a) $f(x) = x^2 - 1$ | d) $f(x) = 2\left(x + 1\right)^2 - 3$ | g) $f(x) = - x + 3 $ |
| b) $f(x) = \left(x - 1\right)^2 + 2$ | e) $f(x) = x + 1 - 3$ | h) $f(x) = \frac{1}{2}\left(x + 4\right)^2$ |
| c) $f(x) = -3 x $ | f) $f(x) = -3\left(x - 2\right)^2 + 1$ | |

3. Match each graph to one of the following functions.

a) $f(x) = -x^2 - 1$

d) $f(x) = x^2 + 2x + 1$

g) $f(x) = x^2 - 2x$

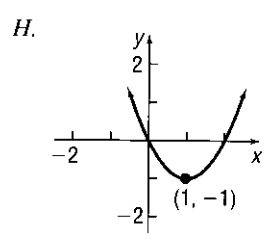
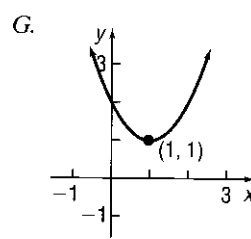
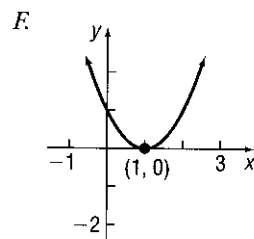
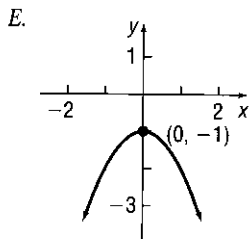
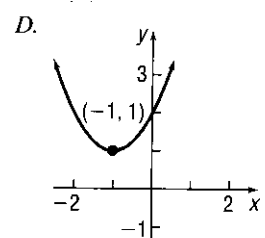
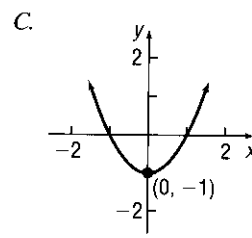
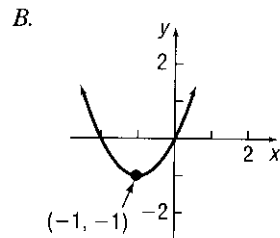
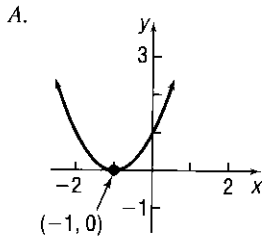
b) $f(x) = x^2 - 2x + 1$

e) $f(x) = x^2 - 2x + 2$

h) $f(x) = x^2 + 2x + 2$

c) $f(x) = x^2 + 2x$

f) $f(x) = x^2 - 1$



4. Write the function in the form $f(x) = a(-h)^2 + k$ and graph it using transformation techniques.

a) $f(x) = \frac{1}{4}x^2$

d) $f(x) = 2x^2 - 4x + 1$

b) $f(x) = \frac{1}{4}x^2 - 2$

e) $f(x) = -x^2 - 2x$

c) $f(x) = x^2 + 4x + 2$

f) $f(x) = \frac{1}{2}x^2 + x - 1$

5. For the given functions f and g , find the following functions and state the domain of each.

a) $f + g$

b) $f - g$

c) $f \cdot g$

d) $\frac{f}{g}$

1) $f(x) = 3x + 4$; $g(x) = 2x - 3$

4) $f(x) = 1 + \frac{1}{x}$; $g(x) = \frac{1}{x}$

2) $f(x) = x - 1$; $g(x) = 2x^2$

3) $f(x) = \sqrt{x}$; $g(x) = 3x - 5$

5) $f(x) = \frac{2x+3}{3x-2}$; $g(x) = \frac{4x}{3x-2}$

6. For the given functions f and g , find

a) $f \circ g$

b) $g \circ f$

c) $f \circ f$

d) $g \circ g$

State the domain of each composite function.

1) $f(x) = 2x + 3$; $g(x) = 3x$

5) $f(x) = \frac{x}{x-1}$; $g(x) = \frac{-4}{x}$

2) $f(x) = 3x + 1$; $g(x) = x^2$

6) $f(x) = \sqrt{x}$; $g(x) = 2x + 3$

3) $f(x) = x^2$; $g(x) = x^2 + 4$

7) $f(x) = x^2 + 1$; $g(x) = \sqrt{x-1}$

4) $f(x) = \frac{3}{x-1}$; $g(x) = \frac{2}{x}$

7. Form a polynomial whose zeros and degree are given.

a) Zeros: -1, 2, 3; degree 3

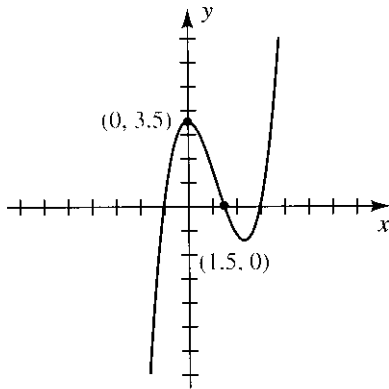
b) Zeros: 4, 3, 0; degree 3

c) -4 and 3 are zeros of multiplicity 2; degree 4

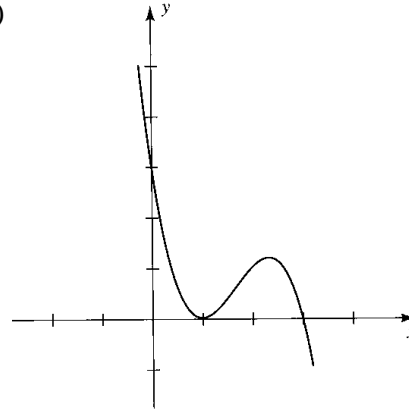
d) -2 and 2 are both zeros of multiplicity 2, 0 is a zero of multiplicity 3; degree 7

8. Find the polynomial whose graph is shown.

a)



b)



9. For each polynomial function:

a) Find the zeros (x-intercepts) and y- intercepts.

b) Multiplicity: Determine whether it touches or crosses at each zero.

c) What is the end behaviour of the function?

d) Use the zeros to find the intervals on which the graph is above or below the x-axis.

e) Graph.

1) $f(x) = x^4 - 4x^2$

2) $f(x) = -x^3 + 3x^2 + 10x$

3) $f(x) = \frac{1}{6} \left(x+2 \right) \left(x-3 \right) \left(x-4 \right)$

4) $f(x) = x^2 \left(x+2 \right) \left(x-1 \right) \left(x-2 \right)$

5) $f(x) = 4x^5 + 12x^4 + 9x^3$

6) $f(x) = x^3 + 2x^2 - 5x - 6$

7) $f(x) = x^4 + x^3 - 3x^2 - x + 2$

8) $f(x) = x^4 + 7x^3 + 13x^2 - 3x - 18$

9) $f(x) = x^6 - 4x^5 + 5x^4 - 5x^2 + 4x - 1$

10. Sketch the graph of the following functions (follow all the steps given in class):

1) $R(x) = \frac{3}{x-4}$

2) $R(x) = \frac{-3x}{x+2}$

3) $R(x) = \frac{-4}{x-2}$

4) $R(x) = \frac{x-3}{x^2-1}$

5) $R(x) = \frac{2x^2-2x-4}{x^2+x-12}$

6) $R(x) = \frac{-x^2-x+6}{x^2+3x-4}$

11. Solve the following systems of equations.

a) $\begin{cases} 2x+3y=2 \\ x-2y=8 \end{cases}$

c) $\begin{cases} 3x+4y=3 \\ x-2y=-4 \end{cases}$

e) $\begin{cases} 2x-3y=5 \\ -6x+9y=12 \end{cases}$

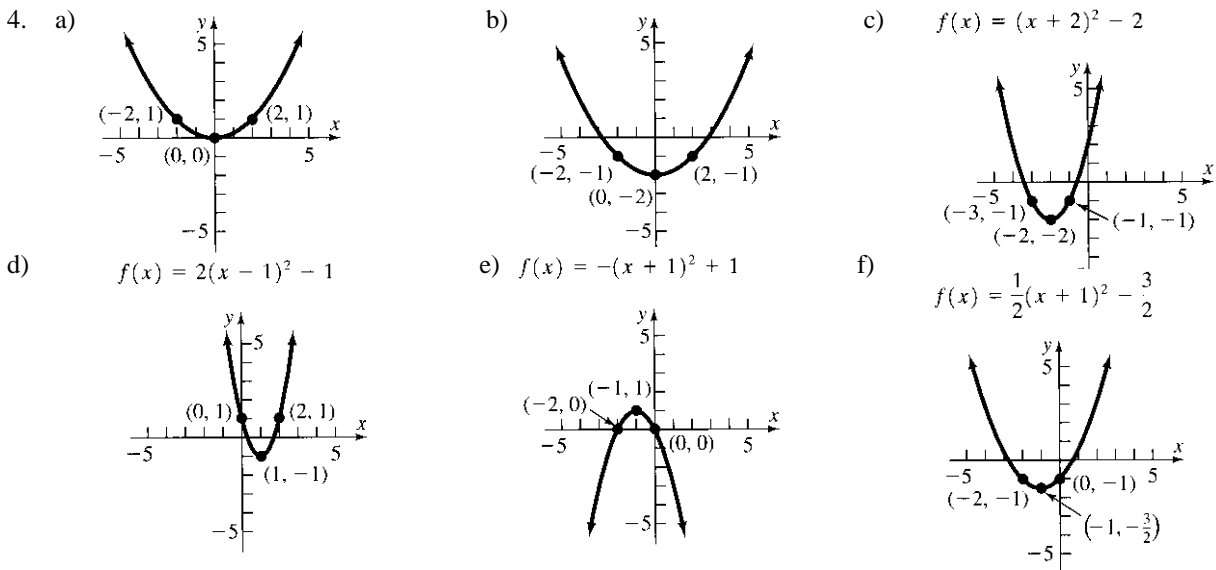
b) $\begin{cases} 2x+5y=16 \\ 3x-7y=24 \end{cases}$

d) $\begin{cases} 2y-5x=0 \\ 3y+4x=0 \end{cases}$

f) $\begin{cases} 3x-4y=2 \\ -6x+8y=-4 \end{cases}$

ANSWERS

- Use the previously downloaded (wzgrapher_e) to check your answers.
- Use the previously downloaded (wzgrapher_e) to check your answers.
- Use the previously downloaded (wzgrapher_e) to check your answers.



5. 1) (a) $(f + g)(x) = 5x + 1$; All real numbers (b) $(f - g)(x) = x + 7$; All real numbers
 (c) $(f \cdot g)(x) = 6x^2 - x - 12$; All real numbers (d) $\left(\frac{f}{g}\right)(x) = \frac{3x + 4}{2x - 3}$; $\left\{x \mid x \neq \frac{3}{2}\right\}$
- 2) (a) $(f + g)(x) = 2x^2 + x - 1$; All real numbers (b) $(f - g)(x) = -2x^2 + x - 1$; All real numbers
 (c) $(f \cdot g)(x) = 2x^3 - 2x^2$; All real numbers (d) $\left(\frac{f}{g}\right)(x) = \frac{x - 1}{2x^2}$; $\{x \mid x \neq 0\}$
- 3) (a) $(f + g)(x) = \sqrt{x} + 3x - 5$; $\{x \mid x \geq 0\}$ (b) $(f - g)(x) = \sqrt{x} - 3x + 5$; $\{x \mid x \geq 0\}$
 (c) $(f \cdot g)(x) = 3x\sqrt{x} - 5\sqrt{x}$; $\{x \mid x \geq 0\}$ (d) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{2x - 5}$; $\left\{x \mid x \geq 0, x \neq \frac{5}{2}\right\}$
- 4) (a) $(f + g)(x) = 1 + \frac{2}{x}$; $\{x \mid x \neq 0\}$ (b) $(f - g)(x) = 1$; $\{x \mid x \neq 0\}$ (c) $(f \cdot g)(x) = \frac{1}{x} + \frac{1}{x^2}$; $\{x \mid x \neq 0\}$
 (d) $\left(\frac{f}{g}\right)(x) = x + 1$; $\{x \mid x \neq 0\}$
- 5) (a) $(f + g)(x) = \frac{6x + 3}{3x - 2}$; $\left\{x \mid x \neq \frac{2}{3}\right\}$ (b) $(f - g)(x) = \frac{-2x + 3}{3x - 2}$; $\left\{x \mid x \neq \frac{2}{3}\right\}$
 (c) $(f \cdot g)(x) = \frac{8x^2 + 12x}{(3x - 2)^2}$; $\left\{x \mid x \neq \frac{2}{3}\right\}$ (d) $\left(\frac{f}{g}\right)(x) = \frac{2x + 3}{4x}$; $\left\{x \mid x \neq 0, x \neq \frac{2}{3}\right\}$

6. 1) (a) $(f \circ g)(x) = 6x + 3$; All real numbers (b) $(g \circ f)(x) = 6x + 9$; All real numbers (c) $(f \circ f)(x) = 4x + 9$; All real numbers (d) $(g \circ g)(x) = 9x$; All real numbers
- 2) (a) $(f \circ g)(x) = 3x^2 + 1$; All real numbers (b) $(g \circ f)(x) = 9x^2 + 6x + 1$; All real numbers (c) $(f \circ f)(x) = 9x + 4$; All real numbers (d) $(g \circ g)(x) = x^4$; All real numbers
- 3) (a) $(f \circ g)(x) = x^4 + 8x^2 + 16$; All real numbers (b) $(g \circ f)(x) = x^4 + 4$; All real numbers (c) $(f \circ f)(x) = x^4$; All real numbers (d) $(g \circ g)(x) = x^4 + 8x^2 + 20$; All real numbers
- 4) (a) $(f \circ g)(x) = \frac{3x}{2 - x}$; $\{x \mid x \neq 0, x \neq 2\}$ (b) $(g \circ f)(x) = \frac{2(x - 1)}{3}$; $\{x \mid x \neq 1\}$ (c) $(f \circ f)(x) = \frac{3(x - 1)}{4 - x}$; $\{x \mid x \neq 1, x \neq 4\}$
 (d) $(g \circ g)(x) = x$; $\{x \mid x \neq 0\}$ 5) (a) $(f \circ g)(x) = \frac{4}{4 + x}$; $\{x \mid x \neq -4, x \neq 0\}$ (b) $(g \circ f)(x) = \frac{-4(x - 1)}{x}$; $\{x \mid x \neq 0, x \neq 1\}$
 (c) $(f \circ f)(x) = x$; $\{x \mid x \neq 1\}$ (d) $(g \circ g)(x) = x$; $\{x \mid x \neq 0\}$ 6) (a) $(f \circ g)(x) = \sqrt{2x + 3}$; $\left\{x \mid x \geq -\frac{3}{2}\right\}$
 (b) $(g \circ f)(x) = 2\sqrt{x} + 3$; $\{x \mid x \geq 0\}$ (c) $(f \circ f)(x) = \sqrt[3]{x}$; $\{x \mid x \geq 0\}$ (d) $(g \circ g)(x) = 4x + 9$; All real numbers
- 7) (a) $(f \circ g)(x) = x$; $\{x \mid x \geq 1\}$ (b) $(g \circ f)(x) = |x|$; All real numbers (c) $(f \circ f)(x) = x^4 + 2x^2 + 2$; All real numbers
 (d) $(g \circ g)(x) = \sqrt{\sqrt{x - 1} - 1}$; $\{x \mid x \geq 2\}$

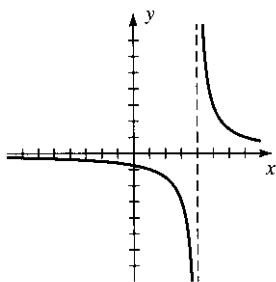
7. a) $f(x) = \left[\left(\left(x + 1 \right) \left(x - 2 \right) \right) \left(x - 3 \right) \right]$ c) $f(x) = \left[\left(\left(x + 4 \right) \right) \left(x - 3 \right) \right]$
 b) $f(x) = x \left[\left(\left(x - 4 \right) \right) \left(x - 3 \right) \right]$ d) $f(x) = x^3 \left[\left(\left(x + 2 \right) \right) \left(x - 2 \right) \right]$

8. a) $f(x) = \frac{7}{9} \left(x + 1 \right) \left(x - \frac{3}{2} \right)$

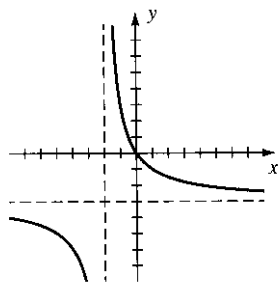
b) $f(x) = - \left(x - 1 \right) \left(x - 3 \right)$

9. Use the previously downloaded (wzgrapher_e) to check your answers.

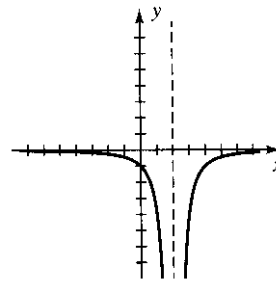
10. 1)



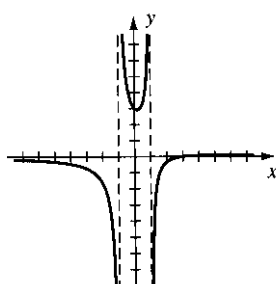
2)



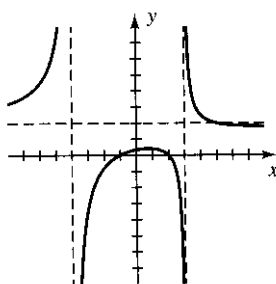
3)



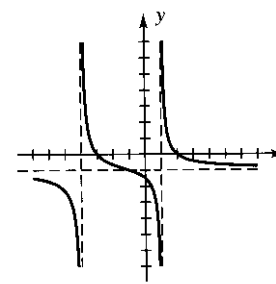
4)



5)



6)



11. a) $\left(-2, \infty \right)$

b) $\left(0, \infty \right)$

c) $\left(-1, \frac{3}{2} \right)$

d) $\left(0, \infty \right)$

e) No solution.

f) $\left(x, \frac{3}{4}x - \frac{1}{2} \right)$